Recursive Form of the Eigensystem Realization Algorithm for System Identification

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An algorithm is developed for recursively calculating the minimum realization of a linear system from sampled impulse response data. The Gram-Schmidt orthonormalization technique is used to generate an orthonormal basis for factorization of the data matrix. The system matrix thus identified is in upper Hessenberg form, which has advantages for the identification of modal parameters including damping coefficients, frequencies, mode shapes, and modal participation factors. It also has the property that once an element of the system matrix is computed, it is never altered as the dimension of the model is increased in the recursive process. Numerical examples are presented for comparison of the recursive and nonrecursive forms of the Eigensystem Realization Algorithm.

Introduction

THERE is a vast body of work on linear system identification, particularly in the fields of structures and controls. In the area of system identification for flexible structures, the identification of modal parameters is a rapidly developing technology.[1–10] Several time-domain and frequency-domain methods have been developed and successfully tested. Many different methods are, in fact, quite similar in the sense that they are mathematically equivalent, from the point of view of system realization theory[2] from the control field.[11] The realization of a linear system is a triplet of matrices $[A,B,C]$ where A is the state matrix, B is the input matrix, and C is the output matrix. The methods developed to date for identification of flexible structures are nonrecursive with only few exceptions, as in Refs. 12 and 13 that use the concept of lattice filter theory to determine the coefficients of the ARMA model. It is difficult to compute the modal parameters from the ARMA model.

The system characteristics of a large flexible structure in space can vary quickly. For example, reorientation of a large antenna on a spacecraft suddenly changes the inertia and stiffness properties. Also, sudden changes of spacecraft shape occur due to temperature gradient induced by shadowing. Attitude and shape control in such applications require real-time computation for quick update of the system model for better control performance. Real-time system identification needs a simple and fast computational procedure that is usually in a recursive form.[14,15]

A simple and fast algorithm is presented for the identification of linear systems from sampled impulse response data. The algorithm, together with certain confidence criteria, represents an alternative version of the Eigensystem Realization Algorithm (ERA).[2,3] The standard version of ERA, which has proved valuable in the structures community for modal parameter identification from test data, operates on a large block data matrix and truncates based on singular values to achieve the final identified model. The algorithm produced here is based on Gram-Schmidt orthonormalization,[16] and recursively builds up the order of the model until the appropriate order is reached. As a result, the recursive version of ERA is very efficient and requires very little storage. It automatically gives the identified system matrix in upper Hessenberg form,[16] which has advantages for eigensystem analysis. It has the desirable property that, given the identification results for any chosen order, the realizations for any lower-order model are immediately obtained by truncation of the system matrices. Examples are given comparing the recursive version to the standard version of ERA.

Background

This section gives some mathematical background, including the continuous-time process and its sampled data, and the basic formulation of a minimum realization. The relationships of modal parameters, such as damping coefficients, natural frequencies, mode shapes, and modal participation factors, to the realized system matrices are included.

The Continuous Process and its Sampled Data

Consider a continuous process that can be represented by finite dimensional linear time-invariant system equations in terms of the system matrices A_c, B, and C:

$$x(t) = A_c x(t) + Bu(t) \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

If the system equations are diagonalizable, Eq. (1) can be transformed into the modal equations

$$q(t) = \Lambda q(t) + \hat{B}u(t) \tag{2a}$$

$$y(t) = \hat{C}q(t) \tag{2b}$$

where $\Lambda = \Psi^{-1}A_c\Psi$ is diagonal, Ψ is a transformation matrix, $\hat{B} = \Psi^{-1}B$, and $\hat{C} = C\Psi$. In the structures field, Λ contains vibration frequencies and damping coefficients, $\hat{B}$ and $\hat{C}$ are called modal participation factors and mode shapes, respectively.

For purposes of identification, the impuse response for zero initial conditions $y^{(j)}(k\,\Delta t) = C\exp(A_c k\,\Delta t)b^{(j)}$ is obtained at

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sample times $k\,\Delta t$, for an impulse at $t = 0$ in the jth input u_j. Here $b^{(j)}$ is the jth column of B, k is an integer, and Δt is the sampling time. The collection of such impulse responses obtained for all j is denoted by

$$Y(k) = C[\exp(A_c\,\Delta t)]^k B = CA^k B \tag{3}$$

where $A = \exp(A_c\,\Delta t)$. Note that $Y(0)$ represents data $Y(0+)$, referring to the time instant immediately after the impulse is applied.

The realization algorithms discussed in the sequel will obtain expressions for A, B, and C, which realize the impulse response sequences according to Eq. (3). Let Ψ be chosen as the matrix that diagonalizes A, i.e.,

$$\begin{aligned}\Psi^{-1}A\Psi &= \mathrm{diag}(\lambda_1,\lambda_2,\ldots,\lambda_n)\\ &= \Psi^{-1}\exp[A_c\,\Delta t]\Psi = \exp[(\Psi^{-1}A_c\Psi)\,\Delta t]\\ &= \exp[\Lambda\,\Delta t]\end{aligned} \tag{4}$$

The system matrices for representation (2) of the continuous system are given in terms of the B, C, and the eigenvalues and eigenvectors of A obtained in the realization algorithm, according to

$$\Lambda_k = \mathrm{diag}(\sigma_1 + i\omega_1,\ldots,\sigma_n + i\omega_n) \tag{5a}$$

$$\sigma_k = \mathrm{Re}[ln(\lambda_k)/\Delta t] \tag{5b}$$

$$\omega_k = \mathrm{Im}[ln(\lambda_k)/\Delta t] \tag{5c}$$

$$\hat{B} = \Psi^{-1}B \tag{5d}$$

$$\hat{C} = C\Psi \tag{5e}$$

Note that the identified matrix A can be thought of as one of the matrices of a difference equation equivalent to the continuous system (1), but because of the impulse response nature of the data, the identified B matrix is that of the continuous system (1), and *not* the usual input matrix for a discrete system that would be obtained using a zero-order hold.

The Eigensystem Realization Algorithm

Let $H(k)$ be the Hankel matrix

$$H(k) = \begin{bmatrix} Y(k) & Y(k+1) & \ldots & Y(k+s-1)\\ Y(k+1) & Y(k+2) & \ldots & Y(k+s)\\ \vdots & \vdots & \vdots & \vdots\\ Y(k+r-1) & Y(k+r) & \ldots & Y(k+r+s-2)\end{bmatrix} \tag{6}$$

of data associated with the n-dimensional system (1), and pick r and s to be any numbers large enough that the rank of the matrix cannot be increased by increasing r and s. Then the rank of $H(k)$ is n for all k (assuming perfect noiseless data).

Define a controllability matrix P, an observability matrix Q, and matrices E_β, E_m according to

$$P = [B, AB, \ldots, A^{s-1}B] \tag{7a}$$

$$Q = [C^T, (CA)^T, \ldots, (CA^{r-1})^T]^T \tag{7b}$$

$$E_\beta^T = [I_\beta, 0_\beta, \ldots, 0_\beta]^T \tag{7c}$$

$$E_m^T = [I_m, 0_m, \ldots, 0_m]^T \tag{7d}$$

where β and m are the dimensions of the measurement vector $y(t)$ and control vector $u(t)$, and I_β, I_m are identity matrices of order β and m, and 0_β and 0_m are $\beta \times \beta$ and $m \times m$ zero matrices. Then it is clear from direct substitution that

$$Y(k) = E_\beta^T H(k) E_m \tag{8a}$$

$$H(k) = QA^kP \tag{8b}$$

In theory, the realization of the triplet $[A,B,C]$ is not unique. Consider a factorization of the rank n matrix $H(0)$ into a product of two rank n matrices

$$H(0) = QP \tag{9}$$

where Q has n columns and P has n rows. Then the first matrix of the factorization Q can represent the observability matrix, whereas the second matrix P can represent the controllability matrix for some set of state variables, representing the controllable and observable part of the system. Then the Moore-Penrose pseudoinverse $H^\dagger(0)$ is given by

$$H^\dagger(0) = P^\dagger Q^\dagger \tag{10}$$

Because the ranks of P and Q are n, $P^\dagger$ and $Q^\dagger$ can be written by

$$P^\dagger = P^T(PP^T)^{-1}, \qquad Q^\dagger = (Q^TQ)^{-1}Q^T \tag{11}$$

Hence, combination of Eqs. (9–11) yields

$$PH^\dagger(0)Q = PP^\dagger Q^\dagger Q = I_n \tag{12}$$

With these mathematical preliminaries, one can obtain the following realization. Using Eqs. (8) and (11)

$$\begin{aligned}Y(k) &= E_\beta^T QA^kPE_m = E_\beta^T Q(Q^\dagger QA^kPP^\dagger)PE_m\\ &= E_\beta^T Q[Q^\dagger H(1)P^\dagger]^k PE_m\end{aligned} \tag{13}$$

and by comparison to Eq. (3) we see that $Y(k)$ can be realized using

$$A = Q^\dagger H(1)P^\dagger, \qquad B = PE_m, \qquad C = E_\beta^T Q \tag{14}$$

Equation (14) is the basic formulation for the ERA.

One choice of the factorization in Eq. (9) is the singular value decomposition[17] that yields

$$H(0) = USV^T \tag{15}$$

where S is the $n \times n$ diagonal matrix of nonzero singular values, and U and V each have n orthonormal columns of appropriate dimension. Therefore, let

$$P = S^{\frac{1}{2}}V^T, \qquad Q = US^{\frac{1}{2}}$$

Equation (14) becomes

$$A = S^{-\frac{1}{2}}U^TH(1)VS^{-\frac{1}{2}}, \quad B = S^{\frac{1}{2}}V^TE_m, \quad C = E_\beta^TUS^{\frac{1}{2}} \tag{16}$$

This is the standard version of the ERA formulation. An alternative decomposition of $H(0)$ in Eq. (9), based on Gram-Schmidt orthonormalization, is used here to obtain desirable recursive properties for Eq. (14).

Discussion

The usual realization in ERA is that of Eq. (16). Since S is an $n \times n$ matrix, then so is A, and Eq. (16) represents a minimal realization. In using this realization, one generally starts from a Hankel matrix of a significantly larger dimension than one expects for the system, and performs singular-value decomposition on this relatively large matrix in order to pick the order n of the realization.

This paper aims at generating a new version of ERA that successively increases the order of the realization until the proper order n is reached, rather than going beyond the proper order and then truncating the system based on the singular values. Such a recursive approach could have storage and computation-time advantages, although one might expect some degradation in accuracy. One would expect such a recursive algorithm to have sufficiently small computational requirements to be appropriate for use on a personal computer.

In this paper, we consider successively increasing the number of columns in the Hankel matrix, maintaining throughout a number of rows greater than the expected order of the system and greater than the number of columns treated. It is necessary to have a method of monitoring the change in the rank of the Hankel matrix as columns are added. Then, when the rank no longer increases, one has arrived at the system order n. We choose to do this using a Gram-Schmidt orthonormalization.

Recursive Orthonormalization and the Pseudoinverse

This section develops certain mathematical results used to generate the identification algorithm. The recursion considered here uses a Hankel matrix with a fixed number of rows, and increases the number of columns one by one. Introduce a single subscripted notation H_j where j denotes the number of columns (which need not be equivalent to an integer number of column partitions). Let the columns of H_j be h_i so that

$$H_j = [h_1,h_2, \ldots ,h_j] \tag{17}$$

Known properties of the Hankel matrix[11] indicate that when the columns represent noiseless measurements of the impulse response of an nth order realization, Eq. (3), the h_i for $i = 1,2, \ldots ,n$, will be linearly independent, but all h_i for $i > n$ can be written as a linear combination of the first n columns.

The Gram-Schmidt orthonormalization of the columns of H_j produces a matrix of orthonormal columns, spanning the same space, according to

$$\hat{H}_j = [\hat{h}_1,\hat{h}_2, \ldots , \hat{h}_j] \tag{18a}$$

$$\bar{h}_i = h_i - \sum_{k=1}^{i-1} (h_i^T\hat{h}_k)\hat{h}_k \tag{18b}$$

$$\hat{h}_i = \bar{h}_i/|\bar{h}_i| \tag{18c}$$

$$i = 1,2,3, \ldots , j \tag{18d}$$

Statement:

1) Matrix $\hat{H}_j$ can be expressed in the form

$$\hat{H}_j = H_j\Phi_j \tag{19}$$

where Φ_j is a $j \times j$ upper triangular matrix.

2) Expression (19) can be recursively computed as j increases using the following formulations:

$$\Phi_1 = 1/|h_1|, \qquad \hat{H}_1 = h_1/|h_1| \tag{20a}$$

$$\bar{h}_{j+1} = h_{j+1} - \hat{H}_j(\hat{H}_j^T h_{j+1}) \tag{20b}$$

$$\Phi_{j+1} = \begin{bmatrix} \Phi_j & -\Phi_j\hat{H}_j^T h_{j+1}/|\bar{h}_{j+1}| \\ 0 & 1/|\bar{h}_{j+1}| \end{bmatrix} \tag{20c}$$

$$\hat{h}_{j+1} = \bar{h}_{j+1}/|\bar{h}_{j+1}| \tag{20d}$$

$$\hat{H}_{j+1} = [\hat{H}_j, \hat{h}_{j+1}], \qquad j = 1,2,3, \ldots , n-1 \tag{20e}$$

where the recursion ends when the number of columns reaches the order n of the system, after which all additional columns are linearly dependent on $\bar{h}_1,\bar{h}_2, \ldots ,\bar{h}_n$ and $|\bar{h}_{n+1}| = 0$.

3) Matrix H_j can be decomposed into the following form:

$$H_j = \hat{H}_j\Gamma_j \tag{21a}$$

$$\Gamma_j = \begin{bmatrix} |\bar{h}_1| & h_2^T\hat{h}_1 & \ldots & h_j^T\hat{h}_1 \\ 0 & |\bar{h}_2| & \ldots & h_j^T\hat{h}_2 \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ 0 & 0 & \ldots & |\bar{h}_j| \end{bmatrix} \tag{21b}$$

where $\Phi_j = \Gamma_j^{-1}$.

Proof: The sequential operations of Gram-Schmidt can be written in the form

$$[\bar{h}_1,h_2, \ldots , h_j] = [h_1,h_2, \ldots , h_j]\phi_1 D_1$$

$$[\hat{h}_1,\bar{h}_2,h_3, \ldots , h_j] = [\hat{h}_1,h_2, \ldots , h_j]\phi_2 D_2$$

$$[\hat{h}_1,\hat{h}_2,\bar{h}_3,h_4, \ldots , h_j] = [\hat{h}_1,\hat{h}_2,h_3, \ldots , h_j]\phi_3 D_3$$

$$\hat{H}_j = H_j[\phi_1 D_1,\phi_2 D_2,\phi_3 D_3, \ldots , \phi_j D_j] = H_j\Phi_j$$

where

$$\phi_1 = I_j, \qquad D_1 = \mathrm{diag}(1/|h_1|,I_{j-1}) \tag{22a}$$

$$D_k = \mathrm{diag}(I_{k-1}, 1/|\bar{h}_k|,I_{j-k}) \tag{22b}$$

$$\phi_k = I_j - C_k \tag{22c}$$

Here C_k is the $j \times j$ zero matrix with the kth column replaced by

$$[h_k^T\hat{H}_{k-1}, 0, \ldots , 0]^T = [h_k^T\hat{h}_1, h_k^T\hat{h}_2, \ldots , h_k^T\hat{h}_{k-1}, 0, \ldots , 0]^T$$

Hence

$$\Phi_j = \phi_1 D_1 \ldots \phi_j D_j \tag{23}$$

is the product of upper triangular matrices and is, therefore, upper triangular.

Assuming that the $j \times j$ matrix Φ_j has been found, then the above expressions can be used to obtain the $(j+1) \times (j+1)$ matrix Φ_{j+1} according to

$$\begin{aligned} \Phi_{j+1} &= \mathrm{diag}(\Phi_j,1)\phi_{j+1}D_{j+1} \\ &= \mathrm{diag}(\Phi_j,1)[I_{j+1} - C_{j+1}]\ \mathrm{diag}[I_j,1/|h_{j+1}|) \end{aligned} \tag{24}$$

where ϕ_{j+1}, D_{j+2}, and C_{j+1} are the matrices obtained from Eq. (22), with j replaced by $j+1$. Then C_{j+1} becomes the $(j+1) \times (j+1)$ zero matrix, with the last column replaced by $[h_{j+1}^T\hat{H}_j,0]^T$. Performing the product in Eq. (24) produces the Φ_{j+1} of Eq. (20).

To establish part 3, note that from Eq. (23)

$$\Gamma_j = \Phi_j^{-1} = D_j^{-1}\phi_j^{-1} \ldots D_1^{-1}\phi_1^{-1}$$

and that a matrix of the form ϕ_k has as its inverse

$$\phi_k^{-1} = I_j + C_k$$

that is readily verified from $(I_j + C_k)(I_j - C_k) = I_j$ since $C_k^2 = 0$.

Recursive Realization Algorithm

Using the notation of the previous section, replace the $H(0)$ of Eq. (6) by H_j, and distinguish this Hankel matrix from a shifted version H_j^τ used in place of $H(1)$. For simplicity we consider the case of a scalar control, so that $m = 1$, in which case

$$H_{j+1} = [h_1,h_1, \ldots , h_j, h_{j+1}] = [H_j, h_{j+1}] \tag{25a}$$

$$H_{j+1}^\tau = [h_2,h_3, \ldots ,h_{j+1}, h_{j+2}] = [H_j^\tau, h_{j+2}] \tag{25b}$$

Let $P = \Gamma_n$ and $Q = \hat{H}_n$ from Eq. (14), and note that $P^\dagger = \Phi_n$ and $Q^\dagger = \hat{H}_n^T$, the sequence $Y(k)$ can be realized using system matrices

$$A = \hat{H}_n^T H_n^\tau \Phi_n, \qquad B = \Gamma_n E_1, \qquad C = E_\beta^T \hat{H}_n \tag{26}$$

The recursive form of A for this realization is obtained from

$$A_{j+1} = [H_j,h_{j+1}]^T[H_j^\tau,h_{j+2}]\Phi_{j+1}$$

Using Eq. (20) gives

$$A_{j+1} = \begin{bmatrix} A_j & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \Delta A_{j2} \\ \Delta A_{j3} & \Delta A_{j4} \end{bmatrix} \tag{27}$$

where

$$\Delta A_{j2} = (\hat{H}_j^T h_{j+2} - A_j \hat{H}_j^T h_{j+1})/|\bar{h}_{j+1}|$$

$$\Delta A_{j3} = \hat{h}_{j+1}^T H_j^\tau \Phi_j = [0,0,\ldots,0,\hat{h}_{j+1}^T h_{j+1}/|\bar{h}_j|]$$

$$\Delta A_{j4} = \hat{h}_{j+1}^T[h_{j+2} - H_j^\tau \Phi_j \hat{H}_j^T h_{j+1}]/|\bar{h}_{j+1}|$$

$$= [\hat{h}_{j+1}^T h_{j+2} - \Delta A_{j3} \hat{H}_j^T h_{j+1}]/|\bar{h}_{j+1}|$$

The dimensions of A_j, ΔA_{j2}, ΔA_{j3}, and ΔA_{j4} are, respectively, $j \times j$, $j \times 1$, $1 \times j$, and 1×1. Note that ΔA_{j3} takes on a particularly simple form because, by construction, $\hat{h}_{j+1}$ is orthogonal to $h_1, \ldots, h_j$. The matrices B and C come from

$$C_{j+1} = [C_j, E_\beta^T \bar{h}_{j+1}], \qquad B_{j+1} = [|\bar{h}_1|, 0, \ldots, 0] \tag{28}$$

where there are j zero elements in B_{j+1}.

To summarize the recursive algorithm, start with

$$\hat{H}_1 = h_1/|h_1|, \qquad A_1 = \hat{h}_1^T h_2/|h_1|$$

Use the next column of the Hankel matrix h_{j+1} and the formulas in Eq. (20) to obtain $|\bar{h}_{j+1}|$, $\hat{h}_{j+1}$, and $\hat{H}_{j+1}$. Equations (27) and (28) then produce A_{j+1}, B_{j+1}, and C_{j+1}. Note that there is no need for an explicit computation of Φ_{j+1}, and that the only quantity that needs to be stored during the computation, besides the model produced, is $\hat{H}_j$. The most time-consuming computation is that of the matrix product $\hat{H}_j^T h_{j+2}$ in the ΔA_{j2} of Eq. (27), and storing it allows one to obtain easily the $\hat{H}_{j+1}^T h_{j+2}$ of the $\bar{h}_{j+2}$ computation, and the $\Delta A_{(j+1)2}$ computation at step $j+1$. At each step j, a decision must be made as to whether appending column h_{j+1} to the Hankel matrix H_j to produce H_{j+1} has increased the rank of the matrix. If not, the current value of j is the dimension n of the realization. In the no-noise case, this situation is recognized by having $|\bar{h}_{j+1}| = 0$. When noise is present, the size of $|\bar{h}_{j+1}|$, which is generally in descending order, is used to make a judgment about the proper order n of the model. Once A_n, B_n, C_n are determined, the continuous time realization (2) is obtained by finding the eigenvalues λ_k of A_n, and the matrix of eigenvectors Ψ of A_n, and using these in Eq. (5).

This algorithm has the following desirable properties:

1) Once an element of the A_j and C_j matrices is computed, it is never altered as the dimension of the model is increased from j to $j+1$; in addition, the nonzero element of B_j is fixed for all j.

2) The algorithm is extremely fast and simple, requiring only a few inner products and a couple of matrix products at each step j.

3) At each step j, only $j+2$ scalars are computed and appended to A_j to produce A_{j+1}, and only β scalars are computed and appended to C_j to produce C_{j+1}. Therefore, the number of computations required grows relatively slowly with the order of the model.

4) When significant noise is present in the data, there can be some ambiguity concerning the proper choice of the realization order n. Given the realization for a value of n, the special structure of this recursive algorithm mentioned in item 3 allows one to obtain all lower-order realizations immediately by truncation of rows and columns.

5) The algorithm is recursive, requiring knowledge of only $\hat{H}_j$ and the next two columns of data in the Hankel matrix, in order to obtain the model of order $j+1$ from that of order j. There is no need to compute or store any auxiliary matrices such as Φ_j.

6) Because of the zero elements of ΔA_{j3} in Eq. (29), the identified system matrix A_n is in upper Hessenberg form. This special form allows one to use particularly efficient eigenvalue and eigenvector routines to obtain the continuous system realization, Eq. (2).

7) Let us repeat here the original purpose of generating a recursive algorithm. The recursion allows one to increase the order of the model until the proper order is reached, rather than starting from a much larger dimension matrix and then decreasing the order to that of the system. This offers savings in computer storage and time in comparison to the more usual ERA realization. To accomplish this, and obtain the other desirable properties listed above, we have abandoned the use of singular value decomposition in favor of a Gram-Schmidt procedure, and one therefore expects some degradation in numerical sensitivity to noise.

8) Another recursive algorithm is derived in the Appendix using a similar approach, but it does not exhibit some of the above desirable features including items 1 and 6.

Numerical Examples

Measurement data for a four-mode system were simulated using additive random noise, and then the recursive algorithm (ERA-RC) developed here was used to identify the system modes. The standard ERA algorithm using Eq. (16) was also used on the data and the results compared.

The simulated data were constructed from

$$Y(k) = \sum_{j-1}^{N} a_j \exp(-\zeta_j \Omega_j t) \cos(\omega_j \tau - \theta_j) + n(t)$$

where the parameter choices are given in Table 1; N, the number of modes, is 4, a_j are their amplitudes, ζ_j are the damping

Table 1 Parameters used for simulation[a]

Mode number	Frequency, Hz	Damping ratio, %	Amplitude	Phase
1	1.0	1.0	1.0	0
2	2.0	2.0	1.0	0
3	3.0	3.0	1.0	0
4	4.0	4.0	1.0	0

[a] Sampling rate: 10 Hz.
Noise: uniformly distributed, white. Noise level = 10% of signal level (ratio of standard deviations). Sample mean value 5.86×10^{-3}. Sample standard deviation 0.0982.
Signal: Mean value of first 40 data points 3.57×10^{-2}. Standard deviation of first 40 data points 1.01.

Table 2 Identification results

	Mode number	Frequency, Hz	Damping ratio, %	Accuracy indicator, %
		Case 1		
ERA-RC	1	0.985	1.38	99.2
	2	1.986	2.68	99.3
	3	2.981	2.99	98.5
	4	4.022	3.57	98.5
ERA	1	0.996	0.88	99.9
	2	1.996	1.76	99.9
	3	3.001	2.67	99.9
	4	4.025	3.83	99.6
		Case 2		
ERA-RC	1	1.004	0.71	99.2
	2	2.014	2.13	98.1
	3	3.008	3.32	98.4
	4	3.850	4.96	82.8
ERA	1	1.004	1.00	98.8
	2	2.011	1.61	97.8
	3	3.016	6.38	92.2
	4	3.922	2.48	89.2

ratios, $\omega_j = 2\pi f_j$ where the f_j are the frequencies in Hz, θ_j are the phase angles taken as zero for all modes, and $\Omega_j = \omega_j/(1-\zeta_j^2)^{\frac{1}{2}}$. The $n(t)$ is the random noise taken as uniformly distributed $[-1,1]$ and white, with a noise level of approximately 10%, measured as a ratio of the noise standard deviation to the standard deviation of the signal for the data time steps used in the identification.

The first identification problem considered used a 20×20 symmetric square Hankel matrix. The identification results for a 20th-order realization were identical (at least to the seven digits of the printout) for the ERA and for the recursive algorithm ERA-RC. However, a nonsymmetric 20×20 Hankel matrix, obtained by skipping every two out of three rows, was found to produce different results.

Table 2 gives the results in two additional cases. Case 1 starts with a 20×20 Hankel matrix and produces an eighth-order model. In the case of ERA-RC, this is done by recursively including columns 1–8 of the Hankel matrix, and for ERA the full 20×20 Hankel matrix was used, and all but the first eight singular values were replaced by zeros. Comparison of the identification results with the correct values given in Table 1 shows that for some of the numbers the ERA-RC gave better results, and for others ERA was better. As a general rule, one would expect that the singular value decomposition is numerically more stable than the Gram-Schmidt orthonormalization. Therefore, the ERA results are somewhat closer to the true values, but the difference is not large. The accuracy indicator listed is the output matrix coherence (modal amplitude coherence) described in Refs. 2 and 3. It is a scalar measure of the similarity between the modal amplitude history from the data, and the modal amplitude history generated by the model.

Experience has shown that better results are often obtained when the model order is larger than the true model, since there is a tendency for the noise to contribute to the extra "modes" of the model, resulting in less noise contamination of the true modes. Case 2 in Table 2 produces an identified system matrix that is 16×16, allowing four extra modes for the noise. The results are seen to be significantly closer to the true values, and ERA-RC is seen to compare quite favorably with ERA. There is an additional difference in the computation for Case 2. It uses a Hankel matrix that includes only every third row of the original Hankel matrix, but is still 20×20 in dimension.

The results of these numerical examples suggest that the recursive version of ERA is quite fast and simple, uses very little storage, and produces results with an accuracy approaching that of the standard ERA.

Concluding Remarks

In general, the appropriate order for the system model is unknown. The recursive algorithm developed here allows one to build up to the proper order. In constrast, the standard version of the ERA creates a large dimensional order and then truncates to the appropriate dimension. Recursive procedures may require more computation than their nonrecursive counterparts. However, this algorithm has no loss in computational efficiency, because once the matrix elements of a realization have been computed, they will never be altered in the following recursive steps. The computations also take advantage of the special Hessenberg structure of the system matrix. The recursive form will produce the results somewhat quicker than the nonrecursive version. Future work will be directed toward the recursive identification with arbitrary forcing inputs, and will consider recursion in both columns and rows.

Appendix: Alternative Recursive Realization

Another recursive implementation of Eq. (14) is derived here. When the Hankel matrix is being extended successively column by column, while the rank is being monitored, one then stops the recursion when the right number of columns n is reached to produce a minimal order realization. Let $P = I$ and $Q = H_n$ in Eq. (9). It is immediately seen that $Y(k)$ can be realized using Eq. (14)

$$A = H_n^\dagger H_n^\tau, \qquad B = E_m, \qquad C = E_\beta^T H_n \tag{A1}$$

Statement: For all H_j, $j = 1,2,\ldots,n$, the pseudoinverse $H_j^\dagger = (H_j^T H_j)^{-1} H_j$ of the full rank matrix H_j can be written as

$$H_j^\dagger = \Phi_j H_j^T \tag{A2}$$

Proof: The proof is trivial by making use of Eq. (21), that is

$$\begin{aligned} H_j^\dagger &= (\Gamma_j^T \hat{H}_j^T \hat{H}_j \Gamma_j)^{-1} \Gamma_j^T \hat{H}_j \\ &= (\Gamma_j^T \Gamma_j)^{-1} \Gamma_j^T \hat{H}_j \\ &= \Gamma_j^{-1} \hat{H}_j^T = \Phi_j H_j^T \end{aligned}$$

Let us generate a recursive calculation of the realization in Eq. (A1). The identified B and C of realization (A1) are immediately available

$$B = E_m = E_1, \qquad C = E_\beta^T H_n \tag{A3}$$

and the system matrix A can be written as follows, making use of Eqs. (A1), (A2), (20), and (25):

$$\begin{aligned} A_{j+1} &= H_{j+1}^\dagger H_{j+1}^\tau = \Phi_{j+1} H_{j+1}^T H_{j+1}^\tau \\ &= \begin{bmatrix} \Phi_{j+1} & -\Phi_j \hat{H}_j^T h_{j+1}/|\hat{h}_{j+1}| \\ 0 & 1/|\hat{h}_{j+1}| \end{bmatrix} \begin{bmatrix} \hat{H}_j^T \\ \hat{h}_{j+1}^T \end{bmatrix} [H_j^\tau \; h_{j+2}] \\ &= \begin{bmatrix} A_j & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \Delta A_{j1} & \Delta A_{j2} \\ \Delta A_{j3} & \Delta A_{j4} \end{bmatrix} \end{aligned} \tag{A4}$$

where

$$\begin{aligned} \Delta A_{j1} &= -\Phi_j \hat{H}_j^T h_{j+1} \hat{h}_{j+1} H_j^\tau / |\hat{h}_{j+1}| \\ \Delta A_{j2} &= \Phi_j \hat{H}_j^T [h_{j+2} - h_{j+1} \hat{h}_{j+1}^T h_{j+2} / |\hat{h}_{j+1}|] \\ \Delta A_{j3} &= \hat{h}_{j+1}^T H_j^\tau / |\hat{h}_{j+1}| \\ \Delta A_{j4} &= \hat{h}_{j+1}^T h_{j+2} / |\hat{h}_{j+1}| \end{aligned}$$

The recursion uses Eqs. (20) and (19) to obtain Φ_j, $\hat{H}_j$ starting from the Φ_1, and $\hat{H}_1$ in Eq. (20), and then compute A_{j+1} from Eq. (A4) starting with $A_1 = \hat{h}_1^T h_2/|h_1|$. The recursive realization, Eqs. (27) and (28), is preferred over this alternative realization for two reasons: the nonzero ΔA_{j1} at each step destroys the Hessenberg form, and a lower-order model cannot be obtained immediately by truncating the last columns and rows.

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